

A critical investigation on the discrepancy between the observational and theoretical Red Giant luminosity function ‘Bump’

Santi Cassisi^{1,2} & Maurizio Salaris^{3,4}

¹*Università degli studi de L’Aquila, Dipartimento di Fisica, Via Vetoio, I-67100, L’Aquila, Italy*

²*Osservatorio Astronomico di Collurania, Via M. Maggini, I-64100, Teramo, Italy - E-Mail: cassisi@astrte.te.astro.it*

³*Max-Planck-Institut für Astrophysik, D-85740, Garching, Germany - E-Mail: maurizio@MPA-Garching.mpg.de*

⁴*Centre d’Estudis Avançats de Blanes, C.S.I.C., E-17300, Blanes, Spain*

ABSTRACT

New theoretical evaluations of the RGB luminosity function ‘bump’ and the ZAHB luminosity covering the range of metallicities typical of galactic globular cluster are presented. The variation of the theoretical RGB bump and ZAHB levels due to the metallicity, original helium content, mixing length value, age, mass loss, bolometric corrections, opacities and equation of state adopted in the evolutionary models is also discussed. These new prescriptions have been taken into account for casting light on a longstanding astrophysical problem connected with the Red Giant Branch evolutionary phase, namely the discrepancy between the observational and the theoretical luminosity of RGB bump. A sample of globular clusters with accurate evaluations of the bump luminosity and spectroscopical metallicity determinations has been selected. The Zero Age Horizontal Branch luminosity at the RR-Lyrae instability strip has been evaluated as accurately as possible, and the observational luminosity difference between the RGB bump and the ZAHB has been compared with the theoretical values. It is shown that there is no significant disagreement between observations and canonical stellar models. The possible applications of this result are also briefly discussed.

Key words: stars: evolution – stars: interiors – globular clusters: general

1 INTRODUCTION

The existence of a well developed and populated Red Giant Branch (RGB) is one of the main characteristics of the Color Magnitude Diagrams (CMDs) of galactic Globular Clusters (GCs). While the T_{eff} location of the RGB provides a means to calibrate the mixing length parameter for GC stars (Chieffi, Straniero & Salaris 1995, Salaris & Cassisi 1996 hereinafter Paper I), the Luminosity Function (LF) of the RGB of GCs is a useful tool to test the inner chemical composition stratification, since the hydrogen abundance outside the degenerate helium core sampled by the thin hydrogen burning shell, affects the rate of evolution during the RGB phase. In particular, due to the fact that the shell is extremely narrow in mass (thickness of the order of $0.001\text{--}0.0001 M_{\odot}$), its crossing any discontinuity in the hydrogen profile leads to a temporary drop in the luminosity and therefore to a characteristic ‘bump’ in the observed differential LF.

It has been known for a long time (see e.g. Thomas 1967, Iben 1968, Renzini & Fusi Pecci 1988, Castellani, Chieffi & Norci 1989, Bono & Castellani 1992) that theoretical RGB LFs display a characteristic bump due to the passage of the thin hydrogen burning shell through the composition discontinuity left by the deepest penetration of the convec-

tive envelope. In spite of this, the first clear identification of the RGB bump in the observed LF of GCs is relatively recent; it goes back to the work by King et al. (1985) on 47 Tuc, and only more recently Fusi Pecci et al. (1990, hereinafter FP90 - 11 clusters), Bergbush (1993 - NGC288), Sarajedini & Norris (1994 - 47 Tuc plus other 5 clusters), Sarajedini & Forrester (1995 - NGC6584), Brocato et al. (1996 - NGC5286, NGC6266, NGC6934, NGC6981) have extended the sample of GCs with a detected RGB bump.

The reason for the difficulty in detecting the bump from GCs photometry is the need of very large RGB star samples in order to obtain a firm identification from the observed LF. According to the discussion in FP90 at least 1000–1500 stars in the upper 4 magnitudes of the LF are required for a clear identification of the bump in metal poor GCs; this star sample is larger than the ones currently available for the best studied metal poor clusters. At higher metallicities the extension in luminosity of the bump is larger, and it is also shifted to lower luminosity, in more populated RGB regions, so it is easier to detect.

In order to avoid uncertainties due to the calibration of the observational data and to the cluster distance modulus it is more reliable to consider, when comparing theoretical RGB bump luminosities with the observations, the param-

ter $\Delta V_{\text{hb}}^{\text{bump}} = (V_{\text{bump}} - V_{\text{hb}})$ (FP90) defined as the difference in visual magnitude between the RGB bump and the Zero Age Horizontal Branch (ZAHB) level.

FP90 tried to use the bump luminosity as a standard candle, in order to calibrate the relation between luminosity of the HB and metallicity. Since the bump luminosity depends also on the age of the stellar populations, and both bump and ZAHB are affected by the He abundance, they explored different scenarios about the age of the GC population and the He content. They compared the observed values of $\Delta V_{\text{hb}}^{\text{bump}}$ from their sample with the theoretical ones obtained from RGB models by Rood & Crocker (1989) and unpublished HB models by Rood (transformed to the observational plane by adopting the Bell & Gustafsson 1978 and Kurucz 1979 transformations); they derived that the best agreement was obtained by assuming a constant age ($t=15$ Gyr) and a constant He abundance ($Y=0.23$), or a constant age ($t=15$ Gyr) and an He abundance scaling as $dY/dZ = 3$ (according to Peimbert & Torres-Peimbert 1977). In these two cases the results of the comparison were:

- i) the run of the theoretical relation with respect to the metallicity is in very good agreement with the observations.
- ii) the absolute value of the theoretical relation is 0.4 mag too bright.

A possible interpretation of this discrepancy has been presented by Alongi et al. (1991); they interpreted the detected difference in the zero-point as a limitation of the standard stellar models, and proposed undershooting from the base of the formal boundary of the convective envelope to reconcile theory and observations. More recently Straniero, Chieffi & Salaris (1992) and Ferraro (1992) pointed out that the proper inclusion of the α -element enhancement in the original chemical composition of the GC stars (see e.g. Wheeler et al. 1989) could help in reducing only partially the discrepancy of the zero-point without invoking undershooting, but until now the question remains unsettled.

In this paper we present new theoretical determinations of the RGB bump and ZAHB luminosities covering the range of metallicities typical of galactic GCs, and discuss the influence of different physical and chemical inputs adopted in the model computations on the value of $\Delta V_{\text{hb}}^{\text{bump}}$. Moreover, we reexamine critically the observational values of $\Delta V_{\text{hb}}^{\text{bump}}$, and by comparing theory with available observational data, we will show that actually a significant discrepancy between standard stellar models and observations does not exist.

The plan of the paper is as follows: the theoretical models are presented in section 2, while section 3 deals with the discussion of the observational data and their comparison with the theoretical values of $\Delta V_{\text{hb}}^{\text{bump}}$; conclusions follow in section 4.

2 THEORETICAL DETERMINATION OF THE HB AND RGB BUMP LUMINOSITY LEVEL

In order to compare the observational values of $\Delta V_{\text{hb}}^{\text{bump}}$ with the theoretical prescriptions, we have computed canonical evolutionary models (no diffusion, no undershooting from the base of the convective envelope) of stars with masses of $0.75M_{\odot}$, $0.80M_{\odot}$ and $0.90M_{\odot}$ - covering a range of masses presently evolving along the RGB of GCs corresponding to ages between approximately 22 Gyr and 12 Gyr

- metallicities $Z=0.0001, 0.0003, 0.0006, 0.001, 0.003, 0.006$, and $Y=0.23$. We have interpolated between these models for each metallicity, in order to obtain the value of V_{bump} corresponding to an average age $t=15$ Gyr for the clusters (the choice of this average age comes out from the extensive analysis about GCs ages by Chaboyer, Sarajedini & Demarque 1992, and Salaris, Chieffi & Straniero 1993). Since until now it is not clear if an age spread really exists in the bulk of the GCs population, we prefer to adopt this conservative hypothesis about the age of the clusters, at least for the purposes of this paper (but see also section 4). This corresponds to our reference scenario, in which all the GCs are coeval and with the same He content.

The He core mass and the surface He abundance at the He ignition corresponding to the same age of 15 Gyr have been adopted in order to compute Zero Age Horizontal Branch (ZAHB) models of different total masses for each value of Z . As for the calibration of the superadiabatic envelope convection, the mixing length calibration described in Paper I, obtained by fitting the empirical T_{eff} values of GC RGBs obtained by Frogel et al. (1983), has been adopted. The evolutionary tracks and the ZAHB models have been translated to the observational plane by adopting the Kurucz (1992) transformations. The theoretical values of $\Delta V_{\text{hb}}^{\text{bump}}$ have then been computed by considering the difference in V magnitudes between the mean V magnitude of the RGB bump region, and V_{zahb} taken at $\log(T_{\text{eff}}) = 3.85$, corresponding approximatively to the average temperature in the RR Lyrae instability strip.

The theoretical relation $\Delta V_{\text{hb}}^{\text{bump}} - [M/H]$, obtained by means of the models previously described, is our *reference* relation that will be compared with the observations. In Table 1, the value of V_{bump} , V_{zahb} at $\log(T_{\text{eff}}) = 3.85$ and $\Delta V_{\text{hb}}^{\text{bump}}$ corresponding to our reference case, are listed for the different metallicities considered.

All the theoretical models have been computed adopting the FRANEC evolutionary code (see Chieffi & Straniero 1989). The OPAL opacity tables (Rogers & Iglesias 1992, Iglesias, Rogers & Wilson 1992) for $T > 10000K$ and the Alexander & Ferguson (1994) opacities for $T < 10000K$ have been used. Both high and low temperature opacity tables are computed adopting the solar heavy elements distribution (Grevesse 1991). The electronic conduction is treated according to Itoh et al. (1983). The equation of state (EOS) by Straniero (1988) has been used, supplemented by a Saha EOS at lower temperatures, as described by Chieffi & Straniero (1989).

Before starting the comparison between our theoretical determinations for $\Delta V_{\text{hb}}^{\text{bump}}$ and the observations, we have also analyzed in detail (computing additional stellar models) the dependence of this quantity on other different physical and chemical inputs adopted in computing stellar models and on the transformations used to transfer the models from the theoretical plane to the observational one. Whereas in literature it is easy to find a lot of exhaustive investigation on the influence on the luminosity level of the HB due to the various inputs adopted in evolutionary computations (Caloi, Castellani & Tornambé 1978, Dorman, Rood & O'Connell 1993 and references therein), a detailed investigation concerning the properties of the RGB bump and therefore of the $\Delta V_{\text{hb}}^{\text{bump}}$ is, till now, still lacking. In the following we

Table 1. Luminosity level of the Horizontal branch at $\log(T_{\text{eff}}) = 3.85$, of the RGB bump and $\Delta V_{\text{hb}}^{\text{bump}}$ obtained by adopting our reference models (see text), $Y=0.23$ and $t=15$ Gyr.

$[M/H]$	M_V^{zahb}	M_V^{bump}	$\Delta V_{\text{hb}}^{\text{bump}}$
-2.348	0.557	-0.332	-0.889
-1.871	0.636	-0.051	-0.687
-1.569	0.679	0.182	-0.497
-1.347	0.715	0.355	-0.360
-0.869	0.823	0.854	0.031
-0.567	0.941	1.332	0.391

Figure 1. *Top:* Absolute luminosity of field and globular cluster RR-Lyrae stars versus metallicity provided by Clementini et al. (1995). The luminosity level of the ZAHB at the average temperature of the instability strip for two different assumptions concerning the equation of state are also displayed. The solid line was evaluated taking into account the EOS provided by Straniero (1988), whereas the dotted line is based on the EOS recently provided by the OPAL group; *Bottom:* the same as the top panel but the absolute luminosities of the RR Lyrae variables have been estimated by adopting a different value for the conversion factor p (i.e. the parameter used for transforming the radial velocities into pulsational velocities).

will discuss the variation of $\Delta V_{\text{hb}}^{\text{bump}}$ induced by changes of the following parameters:

- 1) the metallicity and the distribution of the heavy elements;
- 2) the abundance of Helium Y ;
- 3) the mixing length parameter;
- 4) the age of the stellar population;
- 5) mass loss during the RGB evolution;
- 6) the equation of state;
- 7) the opacity;

8) bolometric corrections.

2.1 Metallicity and heavy elements distribution.

The luminosity of the RGB bump is strongly affected by a variation of the global amount of heavy elements. As a matter of fact a metallicity increase causes a decrease of the luminosity level of the bump. This occurrence is due to the larger extension in mass of the convective envelope during the first dredge up, related to the larger opacity and then, larger values of the radiative gradient with respect to the adiabatic one. Therefore the H discontinuity, located deeper and deeper with increasing metallicity, is reached by the H shell burning earlier during the RGB evolution. Fitting the values of V_{bump} in Tab. 1 as a function of $[M/H] = \log(M/H)_{\text{star}} - \log(M/H)_{\odot} \approx \log(Z) + 1.65$ (where M here is the global metal abundance and Z the global heavy elements fraction) for $t=15$ Gyr yields:

$$M_V^{\text{bump}} = 2.212 + 1.768 \cdot [M/H] + 0.294 \cdot [M/H]^2 \quad (1)$$

with a r.m.s.=0.02 mag.

Also the luminosity of the ZAHB is strongly affected by the heavy elements abundance, due to the variation of the He core mass at the flash with the metallicity. The higher the metallicity, the lower the He core mass and the ZAHB luminosity. From our reference models we derived the following relation between M_V^{zahb} at $\log(T_{\text{eff}}) = 3.85$ and $[M/H]$:

$$M_V^{\text{zahb}} = 1.129 + 0.388 \cdot [M/H] + 0.063 \cdot [M/H]^2 \quad (2)$$

with a r.m.s.=0.011 mag.

In Fig.1 a to b, we compare this relation with the most recent determination of the relation between absolute luminosity of field and globular cluster RR-Lyrae stars and metallicity, published by Clementini et al. (1995), based on new spectroscopical determinations of metallicity and on Baade Wesselink estimates of the absolute magnitudes of the variables. It is worth noting that in Fig.1, only observational data for supposedly unevolved RR Lyrae stars have been considered and therefore they should provide a good estimation of the ZAHB luminosity at the RR Lyrae instability strip. Moreover, Clementini et al. (1995) provide two different tabulations (their Tab.21, columns 5 to 6) concerning the absolute luminosity of their RR Lyrae stars sample, corresponding to two different prescriptions for the conversion factor p between observed and true pulsational velocity. In panels a to b we have displayed their data corresponding respectively to columns 6 and 5 of their Tab. 21. We have derived the global metallicity $[M/H]$ for the stars in their sample by assuming the $[Fe/H]$ and $[\alpha/Fe]$ values given in the paper, and also the errors on the observational determinations of metallicity and luminosity come from their paper. As is evident from the figure, our theoretical relation is in agreement - for both choices concerning the conversion factor p - with the observational data.

From relations (1) and (2) we derive:

$$\Delta V_{\text{hb}}^{\text{bump}} = 1.083 + 1.380 \cdot [M/H] + 0.231 \cdot [M/H]^2 \quad (3)$$

with a r.m.s.=0.023 mag.

These relations, and in particular the third one, that we will use for the comparison with the observations, have been derived from stellar models computed adopting a scaled solar heavy elements distribution. As well known the original chemical composition of GCs stars is characterized by $[\alpha/Fe] > 0$ (see e.g. Wheeler et al. 1989); Salaris, Chieffi & Straniero (1993) have demonstrated that the evolution of low mass low metallicity stars with an α -enhanced heavy elements distribution and a fixed global metallicity $[M/H]$, is very well reproduced by scaled solar models with the same value of $[M/H]$.

We have verified that the same holds if we adopt the recent α -enhanced OPAL and Alexander & Ferguson molecular opacities (see Salaris et al. 1996). In particular the value of ΔV_{hb}^{bump} , obtained using these opacity tables for a fixed $[M/H]$, is coincident - within 0.01 mag - with the one derived from the scaled solar models with the same $[M/H]$.

2.2 The Helium abundance.

The influence of a variation of the original He content on the value of ΔV_{hb}^{bump} has been also tested. As well known, increasing the original He content increases the ZAHB luminosity, due to the more efficient energy generation in the H-burning shell. At the same time, also the level of the bump is shifted to higher luminosity, and the net effect is a slight reduction of the ΔV_{hb}^{bump} . For each $\Delta Y = +0.01$ we derive from our models a reduction of 0.011mag of the ΔV_{hb}^{bump} .

2.3 The mixing length parameter.

The mixing length parameter ml is one of the free parameter which enters in stellar computations. However, as already discussed in Paper I, it is usually constrained by the requirement that the solar radius and the observed colors of red giant stars have to be correctly reproduced. In Paper I we have demonstrated that the value of ml obtained by reproducing the T_{eff} of the Sun may not be suitable also for reproducing the T_{eff} of the GCs RGB. For this reason we have performed the ml calibration for our evolutionary tracks of metal poor low mass stars by reproducing the observational T_{eff} of GCs RGBs as derived by Frogel et al. (1983).

In order to illustrate the dependence of the bump luminosity on the value of the mixing length, we have reexamined the evolutionary tracks discussed in Paper I, computed for different values of ml . We find that $\frac{\Delta V_{hb}^{bump}}{\Delta ml} \approx -0.27mag$. Since we have verified that the level of the ZAHB in the RR Lyrae stars region is not influenced (as it is well known) by a variation of ml , it is obtained that the value of ΔV_{hb}^{bump} is decreased by ≈ 0.04 mag for a variation by $+0.15$ of ml .

It is worth noting that a variation of ml of about 0.15 is large enough to compromise the agreement between theoretical evolutionary models and the T_{eff} of GCs RGB stars. In Paper I we derived a relation between $[M/H]$ and T_{eff} of the GCs RGB by adopting the T_{eff} determinations by Frogel et al. (1983); this relation has been used for calibrating all the stellar models presented in this paper. The dispersion of the observational points around this relation is of the

order of 100K (see the discussion in Paper I), and since this dispersion corresponds to a variation of the ml by around ± 0.10 , we can assume this quantity as an estimate of the maximum uncertainty associated to the calibrated value of ml . Taking into account this indetermination, the variation of the theoretical values of ΔV_{hb}^{bump} results to be less than ± 0.03 mag, that is quite negligible.

2.4 The age of the stellar population.

The location of the bump on the RGB of a stellar track depends on the mass of the model, i.e. on the age of the stellar system in which that star is now evolving (see Straniero & Chieffi 1991), while the ZAHB luminosity is practically independent on the age, at least in the age range spanned by the GCs. From the evolutionary tracks of $0.75M_{\odot}$ - $0.8M_{\odot}$ - $0.9M_{\odot}$ models, we derive for each metallicity an increase of ΔV_{hb}^{bump} by almost 0.024 mag for an increase of 1 Gyr in the age of the clusters.

2.5 The effect of the mass loss on the RGB.

As it is well known, all the low mass stars during their evolution along the RGB experience the phenomenon the mass loss. Mass loss during the RGB is also required to obtain the correct HB morphology of GCs. However mass loss does not affect at all the main evolutionary properties of the stars, as for instance the helium core mass at the He flash and the location in T_{eff} of the RGB. This occurrence is really correct if "conventional" assumptions are made on the efficiency of this phenomenon. In fact, it has been shown (Castellani & Castellani 1991, D' Cruz et al. 1995) that, assuming a very high efficiency of the mass loss mechanism, it could be possible to "obtain" stellar models whose evolution does not follow the prescriptions of the canonical stellar evolution theory.

We have tested if mass loss could affect in some way the bump luminosity on the RGB. For this aim, some evolutionary tracks have been computed adopting various assumptions on the mass loss efficiency. Let us remember that the mass loss phenomenon is usually parametrized in stellar computations the Reimers (1975) formula, in which appears a free parameter η . According to various authors (see for instance Renzini & Fusi Pecci 1988) to finely reproduce the distribution of stars on the HB of the bulk of galactic globular cluster - except the GCs showing HB blue tails in their CMDs it is necessary to adopt for such a parameter a value around 0.3 - 0.4. The evolutionary tracks corresponding to the same $0.8M_{\odot}$ model computed without mass loss in a case and assuming a large efficiency ($\eta = 1.0$) for the mass loss are plotted in Fig.2. One can easily notice that the effect of the mass loss on the bump is absolutely negligible. It is also worth noticing that this result has been obtained assuming a very strong amount of mass loss, as it is confirmed by the occurrence that the star is forced to leave off the RGB before igniting the He central burning. Therefore one can safely assume that, in the range of efficiency of the mass loss phenomenon in real RGB stars, the influence of this mechanism on the bump luminosity is quite negligible.

2.6 The equation of state.

Figure 2. The H-R diagram for a $0.8M_{\odot}$ stellar model computed under two different assumptions concerning the efficiency of the mass loss phenomenon. In the inset the effect of mass loss on the evolutionary tracks in the RGB bump region is shown.

A new equation of state suitable for stellar evolutionary computations has been very recently provided by Rogers, Swenson & Iglesias (1996) (OPAL EOS). Therefore we have decided to test if theoretical values of $\Delta V_{\text{hb}}^{\text{bump}}$ are modified when using this new physical input in the evolutionary models computations. In this case we have supplemented the OPAL EOS (in the regions not covered by the tables), with a Saha EOS (for $T < 5000\text{K}$) and the Straniero (1988) EOS, as described in Salaris, Degl’Innocenti & Weiss (1996).

The use of the OPAL EOS in evolutionary computations has led to a revision of the age of the GCs with respect to determinations obtained by means of stellar models computed using a different EOS (as in Chaboyer et al. 1992 and Salaris et al. 1993). Recent works by Chaboyer & Kim (1995, who find a reduction by 6-7% in the ages derived using $M_V(TO)$ and an average age of 13 Gyr for a sample of 40 clusters), Mazzitelli, D’Antona & Caloi (1995), Salaris et al. (1996), show that the average age of the GCs should be around 12-13 Gyr, significantly lower than the average age obtained in previous works. This result is very important since, as it is well known, the determination of the GCs age is a fundamental tool to investigate the galactic formation mechanism and the age of the Universe. The discussion of these results is out of the goals of the present work, but due to the effect of the age on the RGB bump location, we have to make some realistic assumptions concerning the adopted value for the age of the GCs. Using the same criteria adopted to obtain our reference models, in computing the theoretical values of $\Delta V_{\text{hb}}^{\text{bump}}$ by using the OPAL EOS we have therefore chosen an average age of 12 Gyr for the

GCs.

Once calibrated the mixing length parameter on the observational data by Frogel et al. (1983), we have computed a set of evolutionary tracks as described for our reference case; ZAHB luminosities result to be about 0.055 mag higher than for the models computed with the Straniero EOS (see Fig.1) and $\Delta V_{\text{hb}}^{\text{bump}}$ values are 0.04-0.05 mag lower than the ones obtained adopting our reference scenario and the Straniero EOS. When adopting an age of 15 Gyr also for the models computed with the OPAL EOS, $\Delta V_{\text{hb}}^{\text{bump}}$ values 0.03 mag higher than the reference values are obtained.

2.7 The opacity.

The bump luminosity, being related to the position of the H discontinuity produced by the convective envelope during its deeper penetration, depends strongly on the opacity evaluation for temperatures of the order of one million of degrees, i.e. the temperature at the bottom of the convective envelope. This occurrence has been tested in the past when changing the opacity tables from the Cox & Stewart (1970) and Cox & Tabor (1976) to the most reliable Los Alamos opacity library (hereinafter LAOL; Huebner et al. 1977) the brightness of the bump decreased by $\simeq 0.2\text{mag}$; the change from the LAOL to the OPAL opacities causes a much smaller reduction of the bump luminosity, by $\simeq 0.07\text{mag}$. Now the opacity evaluations are more accurate than in the past, both in the high temperature and in the low temperature region (see e.g. Paper I for a comparison between three recent sets of low temperature opacities). So one can be hopefully confident that there is not much room for a significative variation in the bump brightness as due to a forthcoming generation of updated opacity libraries. For instance, in very recent time, a big effort has been made to improve the accuracy of the OPAL opacity evaluations, by increasing the number of elements taken into account in the metal mixture (Roger & Iglesias 1995 - 21 elements mixture). The effect of this last generation of the OPAL opacities on the bump luminosity has been also checked. As a result, there are not significative variations of $\Delta V_{\text{hb}}^{\text{bump}}$.

2.8 Bolometric corrections.

Our theoretical evolutionary tracks have been transformed into the observational plane by adopting the Kurucz (1992) transformations. In order to check the sensitivity of $\Delta V_{\text{hb}}^{\text{bump}}$ to different sets of bolometric corrections adopted, we have also used the transformations by Buser & Kurucz (1992) supplemented with the Buser & Kurucz (1978) ones for $T > 6000\text{K}$, as described in Salaris et al. (1996). The same values of $\Delta V_{\text{hb}}^{\text{bump}}$ - within 0.01 mag - have been obtained at each metallicity.

3 THEORY VERSUS OBSERVATIONS.

As discussed in the first section, RGB bumps have been detected only in a few globular clusters. To perform a meaningful comparison between theory and observations, we have to consider clusters with a quite clear detection of the bump, and with an accurate determination of the metallicity (taking into account also the enhancement of the α -elements), since the $\Delta V_{\text{hb}}^{\text{bump}}$ is strongly dependent on the

adopted global metallicity. For this reason only clusters with high resolution spectroscopical determinations of the photospheric abundances of Fe and α elements have been taken into account. In Tab.2 the seven clusters considered, the adopted values of $[M/H]$ (these values come from Salaris & Cassisi 1996, where determinations of $[\alpha/Fe]$ and $[Fe/H]$ are collected for a sample of 22 globular clusters), the values of V_{zahb} , V_{bump} , and $\Delta V_{\text{hb}}^{\text{bump}}$ (with its associated observational error) are reported. The sources of the photometric data are given in the discussion of the individual clusters (see below). Five clusters of this GCs sample are very poorly populated or not populated at all in the RR Lyrae stars region (NGC6397, NGC6752, M79, 47 Tuc and NGC6352), and the determination of the ZAHB luminosity in the instability strip is not straightforward. The values of V_{zahb} reported in Tab.2 have been carefully determined by adopting different procedures, depending on the morphology of the cluster HB.

In the case of M3 and M5, whose CMDs display a very well populated HB in the RR Lyrae region, we have adopted the mean luminosity of the RR Lyrae variables ($\langle V_{\text{RR}} \rangle$) as provided in the original works (see below). Clearly $\langle V_{\text{RR}} \rangle$ does not represent the value of the ZAHB luminosity, that has to correspond to the lower envelope of the observed HB stars distribution (see e.g. Sandage 1990); thus, for obtaining the ZAHB level one has to correct the $\langle V_{\text{RR}} \rangle$ value taking into account the thickness of the observed HB. Carney, Storm & Jones (1992) provide a relation between V_{zahb} and $\langle V_{\text{RR}} \rangle$ derived by using the data published by Sandage (1990), who carefully studied the vertical distribution of HB stars in various GCs. They used a sample of 8 clusters to derive the following relation:

$$V_{\text{zahb}} = \langle V_{\text{RR}} \rangle + 0.05[Fe/H] + 0.20 \quad (4)$$

that gives the ZAHB luminosity as a function of the mean luminosity of the RR Lyrae stars and of the cluster iron content. We have performed the same kind of analysis by using clusters, in the sample considered by Carney et al. (1992), for which spectroscopical determinations of $[Fe/H]$ and $[\alpha/Fe]$ (see Paper I) are available, thus obtaining the following relation:

$$V_{\text{ZAHB}} = \langle V_{\text{RR}} \rangle + 0.04[M/H] + 0.15 \quad (5)$$

Relation (5) has been used in order to correct the values of $\langle V_{\text{RR}} \rangle$ for obtaining the ZAHB level. In each case this formula has been used, we have verified that the resulting luminosity matches the lower envelope of the HB stellar population (see Fig. 3a,b).

In the case of NGC6397, NGC6752 and M79 which are characterized by blue HBs, we have followed the same procedure used by other authors (see Buonanno et al. 1986, Alcaïno et al. 1987, Ferraro et al. 1992) in order to derive their ZAHB luminosity. We have considered a reference cluster with the same metallicity but with an HB populated in the blue and in the RR Lyrae region, and its CMD has been shifted in such a way that the blue part of its HB sequence overlaps the blue HB of the cluster we are studying, in order to form a unique sequence (see Fig. 3c-e). In this way the ZAHB level of the cluster with the blue HB has been obtained from the reference cluster, after the correction for

the relative luminosity shift. When it has been selected as reference cluster a GC with the appropriate metallicity, and if the CMD shift has been correctly done, the two RGBs have also to be perfectly overlapped (see Fig. 3c-e), and the horizontal shift applied to the reference cluster has to correspond to the reddening difference between the two clusters.

As for 47 Tuc and NGC6352 a procedure very similar to that described in Fullton et al. (1995) has been adopted. The HBs of these two clusters are populated only on the red side of the instability strip, so no estimate of the ZAHB level at the region of the RR-Lyrae stars is available. Moreover, the value of the ZAHB at the red side can not be used as an estimate of the ZAHB luminosity for the RR Lyrae region (see the discussion in Castellani, Chieffi & Pulone 1991). In this case we have derived from our ZAHB models (transformed to the observational plane by adopting both the Kurucz 1992 and the Buser & Kurucz 1978, 1992 transformations) for $Z=0.003$ and $Z=0.006$ the difference δ in M_V between the red part of the ZAHB and the point along the ZAHB at $\log(T_{\text{eff}}) = 3.85$. We obtained $\delta = 0.10 \pm 0.05$, and applied this correction to the observational data, in order to perform the comparison with the theoretical luminosities at $\log(T_{\text{eff}}) = 3.85$.

In the following the case of each cluster (in order of increasing metallicity) will be discussed separately:

(i) NGC6397: The luminosity of the bump ($V_{\text{bump}} = 12.60 \pm 0.10$) is derived from FP90 who adopt the photometry by Alcaïno et al. (1987). In the paper by Alcaïno et al. (1987) the mean luminosity of the HB in the RR-Lyrae stars region was obtained by superimposing the CMD of M15 (which is populated in the RR Lyrae region) to that of NGC6397 (see previous discussion). We have followed the same procedure (see Fig. 3c) using the CMD of M68 (Walker 1994), which has a global metal abundance much more similar to that of NGC6397 according to the latest spectroscopical determinations ($[M/H]=-1.78$, see Paper I), and a well populated HB in the variable stars region. Walker (1994) provides $\langle V_{\text{RR}} \rangle = 15.64 \pm 0.01$ for the RR Lyrae stars in M68, to which we have subtracted a quantity $\Delta V = -2.70$, corresponding to the shift we have applied to the M68 diagram in order to match the blue HB of NGC6397. The horizontal shift ($\Delta(B-V)$) that we applied to M68 is $\Delta(B-V)=+0.1$, in good agreement with the reddening difference between these two clusters ($E(B-V)_{\text{M68}} = 0.06 - 0.10$ according to Walker 1994, and $E(B-V)_{\text{NGC6397}} = 0.17 - 0.20$ according to Alcaïno et al. 1987). Estimating an uncertainty of around 0.1 mag due to the procedure previously described, after using relation (5) a value $V_{\text{zahb}} = 13.02 \pm 0.10$ is finally derived.

(ii) NGC5272 (M3): The observational data come from Buonanno et al. (1994). In this paper the authors provide the value of $V_{\text{bump}} = 15.40 \pm 0.05$ and $\langle V_{\text{RR}} \rangle = 15.66 \pm 0.05$. By applying the correction for the ZAHB we obtain $V_{\text{zahb}} = 15.76 \pm 0.05$.

(iii) NGC6752: The bump level ($V_{\text{bump}} = 13.65 \pm 0.05$) comes from FP90 and has been determined by using the photometry by Buonanno et al. (1986). In order to derive the ZAHB level we have superimposed the data of M3 (which has a very well populated RR Lyrae region) to that of NGC6752 (see Fig.3d), since the two clusters have quite the same metallicity (see Tab.2). The ZAHB luminosity of M3

Figure 3. CMDs of the clusters for which the ZAHB luminosity level has been obtained using our relation (5), or superimposing another cluster populated in the RR Lyrae region (see text). In each panel the adopted ZAHB luminosity level and the associated error is indicated. In panel c) the C-M diagram of M68 (filled triangles) is superimposed to that of NGC6397 (open circles); the same in panel d) and e), but with M3 (filled triangles) superimposed respectively to NGC6752 and M79 (open circles).

has been corrected by a quantity $\Delta V = -1.90$ corresponding to the vertical shift applied to its CMD in order to match the data of NGC6752. The horizontal shift $\Delta(B - V) = +0.05$ applied to M3 agrees very well with the reddening difference between the two clusters ($E(B - V)_{M3} = 0.00 - 0.03$ according to Buonanno et al. 1994, and $E(B - V)_{NGC6752} = 0.02 - 0.06$ according to Penny & Dickens 1986). We derive $V_{zahb} = 13.86 \pm 0.11$, after taking into account an error of about 0.1 mag due to the procedure of superimposing the

two HB sequence, and the error associated to the estimate of the M3 ZAHB level (see above).

It is very interesting to note the agreement between the cluster distance modulus obtained by adopting this value of V_{zahb} together with our relation (2), and the distance modulus determined very recently by Renzini et al. (1996) in a completely independent way. They use the observed White Dwarf (WD) cooling sequence of NGC6752 as a distance indicator, and derive the distance modulus by fit-

Table 2. Luminosity of the bump, of the Horizontal branch at $\log(T_{\text{eff}}) = 3.85$ and $\Delta V_{\text{hb}}^{\text{bump}}$ for the clusters considered.

Name	V_{zahb}	V_{bump}	$\Delta V_{\text{hb}}^{\text{bump}}$	[M/H]
NGC104	14.20	14.55	0.35 ± 0.18	-0.70
NGC1904	16.36	16.00	-0.36 ± 0.12	-1.27
NGC5272	15.76	15.40	-0.36 ± 0.07	-1.31
NGC5904	15.15	14.95	-0.20 ± 0.07	-1.19
NGC6352	15.50	15.86	0.36 ± 0.12	-0.70
NGC6397	13.02	12.60	-0.42 ± 0.14	-1.70
NGC6752	13.86	13.65	-0.21 ± 0.12	-1.28

ting the cluster WD sequence to an empirical cooling sequence constructed using local WDs with well determined trigonometrical parallaxes. Following this procedure they derive $(m - M)_o = 13.05$ with an overall uncertainty less than ± 0.1 mag. By adopting our relation (2) together with $V_{\text{zahb}} = 13.86 \pm 0.11$, and $A_V = 0.12 \pm 0.06$ (see Renzini et al. 1996, Penny & Dickens 1986), we obtain $(m - M)_o = 13.01 \pm 0.13$, in good agreement with the result by Renzini et al (1996).

(iv) NGC1904 (M79): The observational data come from Ferraro et al. (1992). We adopt $V_{\text{bump}} = 16.00 \pm 0.05$ as estimated by the authors. From the observational data we have determined the ZAHB level following the same procedure as described for NGC6397 and NGC6752. Also in this case we have shifted the data of M3 in order to superimpose the two HB sequences (see Fig.3e). The vertical shift applied to M3 is $\Delta V = +0.6$, while no correction to its (B-V) values has been applied; this is in good agreement with the fact that the reddening of the two clusters $(E(B - V))_{\text{M79}} = 0.00 - 0.02$ according to Ferraro et al. 1992) is coincident. The ZAHB luminosity of M79 results to be $V_{\text{zahb}} = 16.36 \pm 0.11$.

(v) NGC5904 (M5): The bump luminosity has been derived from Brocato, Castellani & Ripepi (1995, 1996) ($V_{\text{bump}} = 14.95 \pm 0.05$). From the same authors we have derived an average magnitude for the RR Lyrae stars ($V_{\text{RR}} = 15.05 \pm 0.05$). By applying relation (5) $V_{\text{zahb}} = 15.15 \pm 0.05$ is obtained.

(vi) NGC104 (47 Tuc) and NGC6352: The bump ($V_{\text{bump}} = 14.55 \pm 0.05$) and the ZAHB luminosities ($V_{\text{zahb}} = 14.10 \pm 0.15$) of 47 Tuc come from FP90, who adopted the photometry by King et al. (1985). As for NGC6352 we have used the data from Sarajedini & Norris (1994) obtaining $V_{\text{zahb}} = 15.40 \pm 0.10$ from the fit of the lower envelope of the HB stellar population, and $V_{\text{bump}} = 15.86 \pm 0.05$ as quoted by the authors. Fullton et al. (1995) published recently another photometry for NGC6352, obtained in part from ground based observations and in part from *HST* observations, and they found a zero point difference in the V magnitudes between their work and the results by Sarajedini & Norris, for the 145 stars in common. However, this zero point difference (around 0.16 mag) does not affect at all - as obvious - the differential quantity $\Delta V_{\text{hb}}^{\text{bump}}$.

By applying the correction $\delta = 0.10 \pm 0.05$ due to their red HB (see the previous discussion), we obtain $V_{\text{zahb}} = 14.20 \pm 0.17$ for 47 Tuc and $V_{\text{zahb}} = 15.50 \pm 0.11$ for NGC6352.

In Fig.4, the comparison between the observational values of $\Delta V_{\text{hb}}^{\text{bump}}$ for the seven clusters considered and the

Figure 4. The values of $\Delta V_{\text{hb}}^{\text{bump}}$ versus the global metallicity for all the clusters in our sample. Our *reference* relation (see text) (solid line) and the theoretical prescription obtained using in stellar computations the OPAL EOS (dashed line) are also plotted.

theoretical prescriptions is reported. We have taken into account the observational errors in the determination of such a quantity (see Tab. 2), and assumed an uncertainty of 0.15dex on the spectroscopic estimates of [M/H] (see the discussions in Gratton, Quarta & Ortolani 1986, Gratton & Ortolani 1989 and Kraft, Sneden, Langer & Shetrone 1993). The theoretical relations corresponding to the models computed adopting the Straniero (1988) EOS and the OPAL EOS are displayed. These two relations correspond - as previously discussed - to an age of 15 Gyr (Straniero EOS) and 12 Gyr (OPAL EOS), and they have been computed by adopting $Y=0.23$.

The figure clearly demonstrates the overall agreement between theoretical standard stellar models (computed with the Straniero or the OPAL EOS) and observations, either for the run of $\Delta V_{\text{hb}}^{\text{bump}}$ with respect to [M/H], either for the absolute values of this quantity; all the observational points are fitted within the observational error bars. The worst agreement is obtained for NGC6397, the most metal poor cluster in our sample.

We may therefore conclude that *there is no significant discrepancy between observations and canonical stellar models* computed by adopting updated input physics, under the assumption of a constant age for the GCs, and a constant initial Helium abundance.

4 DISCUSSION AND CONCLUSIONS.

In the two previous sections we have presented new theoretical stellar models computed with updated input physics, with the aim of comparing the theoretical values of $\Delta V_{\text{hb}}^{\text{bump}}$ with the observations. Before comparing this quantity with real clusters, we had to make some assumptions about the age and the original He content of our

Figure 5. As in Fig.4, but the theoretical relation $\Delta V_{\text{hb}}^{\text{bump}} - [\text{M}/\text{H}]$ are now displayed for our *reference* case (solid line), with two different assumptions concerning the cluster age (13Gyr - short dashed line; 17 Gyr - long dashed line) and for the case of an Helium abundance scaling with the metallicity (see text) (dotted line).

models, since the $\Delta V_{\text{hb}}^{\text{bump}}$ is influenced both by the age (a change of the age affects the bump luminosity) and by the value of Y (a variation of the Helium content modifies the ZAHB and bump luminosities). In the conservative hypothesis of coeval GCs with the same original He content ($Y=0.23$), a very good agreement between theory and observations can be derived from Fig.4. This means that standard stellar models can actually reproduce the luminosity levels of ZAHB and bump, and their run with respect to the metallicity; it confirms also that standard stellar models reproduce accurately the Hydrogen profile in the interior of RGB GCs stars. The differences between our results and the conclusions by FP90 are due basically to three different reasons:

- i) the use of updated evolutionary models;
- ii) the adoption of new spectroscopical determinations of $[\alpha/\text{Fe}]$ and $[\text{Fe}/\text{H}]$;
- iii) new observational data.

FP90 have used models by Rood computed with the old Cox & Stewart (1970) and Cox & Tabor (1976) opacities and, as discussed in their paper, it is not taken into account in the computation of their HB models the extra helium brought to the surface during the first dredge up. In addition, as yet discussed in the introduction, they used also a different set of bolometric corrections (Bell & Gustafsson 1978 and Kurucz 1979). The net resulting difference between our theoretical values of $\Delta V_{\text{hb}}^{\text{bump}}$ and the ones used by FP90 (that arises from the differences on both the ZAHB and the Bump levels) ranges from 0.12 up to 0.22 mag (it is higher at higher metallicities). The remaining part of the discrepancy between theory and observations disappears when considering individual high resolution spectroscopical determination

of $[\text{M}/\text{H}]$, including the overabundance of the α elements, and adopting (when they are available) more recent observational data.

As a further step we can now check how the fit presented in Fig.4 is modified by relaxing the hypothesis of coeval GCs or of a constant Y . If we consider a constant Y but an age spread by ± 2 Gyr around our assumed average age of the clusters (see e.g. Chaboyer, Demarque & Sarajedini 1996), the theoretical $\Delta V_{\text{hb}}^{\text{bump}}$ values would be spread by approximately ± 0.05 mag around the line displayed in Fig.4. In Fig.5 two lines corresponding to the $\Delta V_{\text{hb}}^{\text{bump}}$ values obtained for our average age increased and decreased by 2 Gyr are displayed. As for a variation of Y with the metallicity, the effect of a $dY/dZ = 3$ on the $\Delta V_{\text{hb}}^{\text{bump}}$ is shown in Fig.5. It is evident from the figure that the adopted variation of Y with the metallicity has a negligible effect on the theoretical $\Delta V_{\text{hb}}^{\text{bump}}$ values (at most 0.02 mag at $[\text{M}/\text{H}]=-0.6$), and also the assumed spread in the ages of the clusters does not change very much the theoretical $\Delta V_{\text{hb}}^{\text{bump}} - [\text{M}/\text{H}]$ relation, remaining always compatible with the observational data. Because the agreement between our theoretical $\Delta V_{\text{hb}}^{\text{bump}}$ values and the observations, relation (3) can be safely used for deriving the clusters metallicities. By simply differentiating this relation, one obtains that a variation of the metallicity by ± 0.15 dex around $[\text{M}/\text{H}]=-1.3$ corresponds to a variation of $\Delta V_{\text{hb}}^{\text{bump}}$ by ± 0.12 mag. Allowing for an observational error of around 0.12 mag (see for example Tab.2) in the determination of $\Delta V_{\text{hb}}^{\text{bump}}$, and for an error of around 0.1 mag due to uncertainties in the age, He content of the cluster and the mixing length calibration of the theoretical models (see section 2), it would be possible to estimate the metallicity of the cluster with an error of around ± 0.20 dex (if $[\text{M}/\text{H}]$ is around -1.3). It is worth noticing that the luminosity difference between the bump and the Horizontal branch is a metallicity index which relies only on the observation of luminous stars in the GCs and is less dependent on the precise value of the mixing length than others metallicity indicators based on the colors or on the shape of the RGB.

When the metallicity of a cluster is known, it is possible to use the bump luminosity as a standard candle. Relation (3) can be used as a tentative guess to estimate the HB luminosity level in those globular clusters in which the horizontal branch is poorly populated near the RR-Lyrae instability strip or the HB is quite blue or quite red. It could be interesting to test this procedure in evaluating the Helium abundance on the basis of the R method in those clusters for which estimating the HB luminosity level is a thorny problem.

Before concluding we want to stress again that in this investigation we have computed canonical stellar evolutionary models, neglecting other non canonical effects, as, for instance, the Helium (and heavy elements) diffusion. The inclusion of this mechanism in solar models changes, among other quantities, the chemical abundances in the convective envelope and the depth of the convective region with respect to canonical models, improving the agreement with helioseismological data (see, e.g., the discussion in Castellani et al. 1996).

Proffitt & Vandenberg (1991) have studied the evolution of GCs stars taking into account He diffusion, and very re-

cently Castellani et al (1996) have shown that He and heavy elements diffusion does not change appreciably (age differences by less than 1 Gyr) the age of the GCs with respect to canonical evaluations, but at present time an analysis of the influence of the He and heavy elements diffusion on the $\Delta V_{\text{hb}}^{\text{bump}}$ does not yet exist; from preliminary computations at $Z=0.0004$ (Degl'Innocenti 1996, private communication) it results that this quantity is changed by only ≈ 0.02 mag with respect to the canonical value, but in order to test further the stellar models which include diffusion, computations also for higher metallicities have to be performed.

ACKNOWLEDGMENTS

We gratefully thank Giuseppe Bono, Vittorio Castellani, Martijn De Kool and Achim Weiss for helpful discussions and a careful preliminary reading of the manuscript; Scilla Degl'Innocenti, Anna Piersimoni and Oscar Straniero are acknowledged for stimulating discussions on this subject. We also warmly thank Dave Alexander for providing us with his α -enhanced low temperature molecular opacities, and the referee, Brian Chaboyer, for constructive remarks and observations that have improved the level of the paper.

REFERENCES

- Alcaino G., Buonanno R., Caloi V., Castellani V., Corsi C.E., Iannicola G. & Liller W. 1987, *AJ* 94, 917
- Alexander D.R. & Ferguson J.W. 1994, *ApJ* 437, 879
- Alongi M., Bertelli G., Bressan A. & Chiosi C. 1991, *A&A* 244, 95
- Bahcall J.N. & Loeb A. 1990, *ApJ* 360, 267
- Bell R.A. & Gustafsson B. 1978, *A&AS* 34, 229
- Bergbush P.A. 1993, *AJ* 106, 1024
- Bono G. & Castellani V. 1992, *A&A* 258, 385
- Brocato E., Buonanno R., Malakhova Y. & Piersimoni A.M. 1996, *A&A in press*
- Brocato E., Castellani V. & Ripepi V. 1995, *AJ* 109, 1670
- Brocato E., Castellani V. & Ripepi V. 1996, *AJ* 111, 809
- Buonanno R., Caloi V., Castellani V., Corsi C.E., Fusi Pecci F. & Gratton R.G. 1986 *A&AS* 66, 79
- Buonanno R., Corsi C.E., Buzzoni A., Cacciari C., Ferraro F.R. & Fusi Pecci F. 1994, *A&A* 290, 69
- Buser R. & Kurucz R.L. 1978, *A&A* 70, 555
- Buser R. & Kurucz R.L. 1992, *A&A* 264, 557
- Caloi V., Castellani V. & Tornambé A. 1978, *A&AS* 33, 169
- Carney B.W., Storm J. & Jones R.V. 1992, *ApJ* 386, 663
- Castellani M. & Castellani V. 1993, *ApJ* 407, 649
- Castellani V., Chieffi A. & Norci L. 1989, *A&A* 216, 62
- Castellani V., Chieffi A. & Pulone L. 1991, *ApJS* 76, 911
- Castellani V., Ciacio F., Degl'Innocenti S. & Fiorentini G. 1996, preprint
- Chaboyer B. & Kim Y.-C. 1995, *ApJ* 454, 767
- Chaboyer B., Demarque P. & Sarajedini A. 1996, *ApJ* 459, 558
- Chaboyer D., Sarajedini A. & Demarque P. 1992, *ApJ* 394, 515
- Chaboyer B., Kernan P.J., Krauss L.M. & Demarque P. 1995, preprint CITA-95-18
- Chieffi A. & Straniero O. 1989, *ApJS* 71, 47
- Chieffi A., Straniero O. & Salaris M. 1995, *ApJL* 445, 39
- Clementini G., Carretta E., Gratton R., Merighi R., Mould J.R. & McCarthy J.K. 1995, *AJ* 110, 2319
- Cox A.N. & Stewart J.N. 1970, *ApJS* 19, 243
- Cox A.N. & Tabor J.E. 1976, *ApJS* 31, 271
- D'Cruz N., Dorman B., Rood R. & O'Connell R. 1995, *Bull. American Astron. Soc.* 186, 2201
- Dorman B., Rood R.T. & O'Connell R.W. 1993, *ApJ* 419, 596
- Ferraro F.R. 1992, *MemSAIt* 63, 491
- Ferraro F.R., Clementini, G., Fusi Pecci F. & Sortino R. 1992, *MNRAS* 256, 391
- Frogel J.A., Persson S.E. & Cohen J.G. 1983, *ApJS* 53, 713
- Fullton L.K., Carney B.W., Olzewski E.W., Zinn R., Demarque P., Janes K.A., Da Costa G.S. & Seitzer P. 1995, *AJ* 110, 652
- Fusi Pecci F., Ferraro F.R., Crocker D.A., Rood R.T. & Buonanno R. 1990, *A&A* 238, 95
- Gratton R.L. & Ortolani S. 1989, *A&A* 211, 41
- Gratton R.L., Quarta M.L. & Ortolani S. 1986, *A&A* 169, 208
- Grevesse N. 1991, in "Evolution of stars: the photospheric abundance connection", IAU Symp. eds. Michaud G., Tutukov A., p.63
- Huebner W.F., Merts A.L., Magee Jr. N.H. & Argo M.F. 1977, Los Alamos Sci. Lab. Rep. LA-6760-M
- Iben I. Jr 1968, *Nature* 220, 143
- Iglesias C.A., Rogers F.J. & Wilson B.G. 1992, *ApJ* 397, 717
- Itoh N., Mitake S., Iyetomi H. & Ichimaru S. 1983, *ApJ* 273, 774
- King C.R., Da Costa G.S. & Demarque P. 1985, *ApJ* 299, 674
- Kraft R.P., Sneden C., Langer G.E. & Shethrone M.D. 1993, *AJ* 106, 1490
- Kurucz R.L. 1979, *ApJS* 40, 1
- Kurucz R.L. 1992, in Barbuy B., Renzini A. (eds.), IAU Symp. n. 149, "The Stellar Populations of Galaxies", Kluwer, Dordrecht, p. 225
- Mazzitelli I., D'Antona F. & Caloi V. 1995, *A&A* 302, 382
- Peimbert M. & Torres-Peimbert S. 1977, *MNRAS* 179, 217
- Penny A.J. & Dickens R.J. 1986, *MNRAS* 220, 845
- Proffitt C.R. & Vandenberg D.A. 1991, *ApJS* 77, 473
- Reimers D. 1975, *Mem.Soc.Roy.Sci.Liege*, 6^e Ser. 8, 369
- Renzini A. & Fusi Pecci F. 1988, *ARA&A* 26, 199
- Renzini A., Bragaglia A., Ferraro F.R., Gilmozzi R., Ortolani S., Holberg J.B., Liebert J., Wesemael F. & Bohlin R.C. 1996, *ApJ in press*
- Rogers F.J. & Iglesias C.A. 1992, *ApJS* 79, 507
- Rogers F.J. & Iglesias C.A. 1995, in Adelman S.J., Wiese W.L. (eds.), "Astrophysical application of powerful new databases" ASP Conference series, vol.78, p.31
- Rogers F.J., Swenson F.J. & Iglesias C.A. 1996, *ApJ* 456, 902
- Rood R.T. & Crocker D.A. 1989, in Schmidt E.G. (ed.), IAU Coll. 111, "The use of Pulsating stars in Fundamental Problems of Astronomy", Cambridge University Press, p. 103
- Salaris, Chieffi A. & Straniero O. 1993, *ApJ* 414, 580
- Salaris M. & Cassisi S. 1996, *A&A* 305, 858
- Salaris M., Degl'Innocenti S. & Weiss A. 1996, *ApJ* submitted
- Sandage A. 1990, *ApJ* 350, 603
- Sarajedini A. & Norris J.E. 1994, *ApJS* 93, 161
- Sarajedini A. & Forrester W.L. 1995, *AJ* 109, 1112
- Straniero O. 1988, *A&AS* 76, 157
- Straniero O. & Chieffi A. 1991, *ApJS* 76, 525
- Straniero O., Chieffi A. & Salaris M. 1992, *MemSAIt* 63, 315
- Thomas H.-C. 1967, *Z.Ap.* 67, 420
- Walker A.R. 1994, *AJ* 108, 555
- Wheeler J.C., Sneden C. & Truran J.W. 1989, *ARA&A* 27, 279

This paper has been produced using the Blackwell Scientific Publications \TeX macros.









